Energy, Mines and Resources Canada | Energie, Mines et Ressources Canada

CANMET

Canada Centre for Mineral and Energy Technology | Centre canadien de la technologie des minéraux et de l'énergie

BOUNDARY ELEMENT PROGRAM FOR STRESS ANALYSIS AROUND UNDERGROUND EXCAVATIONS

M.K. GANGAL

ELLIOT LAKE LABORATORY

DECEMBER 1982

This document is an unedited interim report prepared primarily for discussion and internal reporting purposes. It does not represent a final expression of the opinion of the Canada Centre for Mineral and Energy Technology (CANMET)

Ce document est un rapport provisoire non révisé et rédigé principalement à des fins de discussion et de communication interne. Il ne représente nullement l'opinion définitive du Centre canadien de la technologie des minéraux et de l'énergie (CANMET)

45 pp

MINERALS RESEARCH PROGRAM
MINING RESEARCH LABORATORIES
DIVISION REPORT MRP/MRL 83-8(TR)

BOUNDARY ELEMENT PROGRAM FOR STRESS ANALYSIS AROUND UNDERGROUND EXCAVATIONS

by

M.K. Gangal*

ABSTRACT

A modified version of a two-dimensional boundary element program for determining the stress distribution and displacements around a long excavation of specified cross-section in a linearly elastic media is described. The report discusses the program modifications and a test example of multiple excavation with complete documentation. Program listings are included.

* Research Scientist, Elliot Lake Laboratory, CANMET, Energy, Mines and Resources Canada, Elliot Lake, Ontario.

CONTENTS

	Page
ABSTRACT	i
INTRODUCTION	1
SIGN CONVENTION AND SYSTEM OF UNITS	2
BOUNDARY ELEMENT PROGRAM	3
Program Modifications	3
Data Input Procedure	5
Printout Interpretation	8
CIRCULAR ARC SEGMENT PROGRAM	11
TEST EXAMPLE	13
ACKNOWLEDGEMENT	15
REFERENCES	19
APPENDIX A - Circular Arc Segment Program Listing	A-20
APPENDIX B - Boundary Element Program Listing	B-24
APPENDIX C - Multiple Excavation Test Example Printout	C-34

FIGURES

No.		
1.	Boundary segments	9
2.	Modelling of circular arc segment	12
3.	Multiple excavation shape for test example	14
4.	Input data to approximate angular corners by circular arc segments	16
5.	Computer printout for the approximation of angular corners by circular arc segments	17
6.	Input for boundary element program	18

INTRODUCTION

The stresses in an undisturbed rock mass are distributed according to the geological history and the weight of the overlaying strata of the rock mass. When an underground excavation is made, the stress distribution in the neighbourhood of excavation is changed. Depending on the rock properties and the size and shape of the excavation the newly developed stresses may be high enough to exceed the strength of the rock and may cause rock failure. The rock failure could be in the form of gradual closure of the excavation, roof falls, slabbing of side walls, etc.

The safe design of support for underground excavations requires the estimation of stress distribution in the neighbourhood of the openings. There are exact solutions in the theory of elasticity for stress distribution around openings for single, simple geometrical shapes, such as those with circular or elliptical cross-sections. However, more complex shaped openings require numerical solutions. A simple boundary element method for stress analyses of long openings using plane strain problem is described by Brady and Bray (1). The boundary element method (2,3) has many advantages over the finite element method, such as the simplicity of data input, which requires only the field stresses and the excavation boundaries, and the low computer memory requirement.

A two-dimensional boundary element program to determine the stresses around a long excavation was developed at Imperial College and Golder Associates, Vancouver and is described in a book by Hoek and Brown (4). This program uses elastic theory which means in practical terms that it is limited to use in excavation design in hard rock. This program requires a large amount of computer memory for larger size element problems and could not be used on the departmental computer. Therefore, some modifications to the program were

necessary. However, many modifications were made and are described in the report. The modified program considerably reduces the memory requirements and can run a larger element size problem on the departmental CDC Cyber 730 computer. The new version of the program also eliminates the guess work for the number of cycles of iteration required to run the problem as needed in the original program. An additional program was written to convert the angular corners to circular arc segments with results directly usable in the modified program. It was necessary to approximate the angular corners to more realistic circular corners and also to obtain stress analyses at corners of the interior mass rock region. The original program did not provide any stress analyses at corner regions. The complete modified program is listed, and the run on a test example is described in detail. The following assumptions are made in the program:

1. The material is homogeneous, isotropic and linearly elastic.
2. The conditions of plain strain are taken. The Y-direction is the principal stress direction at all points in the medium, therefore the axis of opening in the material is parallel to Y-direction.
3. The medium is taken as infinite or closed by a finite external boundary of arbitrary shape.
4. The program is applicable to single and multiple openings. Therefore the medium may contain a number of holes of arbitrary shape.
5. The loading may consist of any combination of uniform field stresses or of uniformly distributed loads on the boundaries. The gravitational loading is simulated by increasing the field stresses with depth.

SIGN CONVENTION AND SYSTEM OF UNITS

The sign convention for stress is compressive - positive. The direction

of the inwards normal is that of a compressive normal force.

σ_1 and σ_3 are taken as major and minor principal stresses, i.e., $\sigma_1 \geq \sigma_3$. This convention is followed even if all principal stresses are not positive. For example, if σ_1 and σ_3 are tensile (i.e., negative), σ_1 would be the smallest principal tensile stress and σ_3 the largest.

The input and output of the program are in S.I. units except the displacements UX and UZ in the printout, which are printed in millimetres for clarity.

BOUNDARY ELEMENT PROGRAM

PROGRAM MODIFICATIONS

A number of modifications were made to the original program during testing of the program and its application to case studies.

1. Iteration Requirements: A large amount of computing time is spent in determining the fictitious loads, which are calculated by solving a set of simultaneous equations. It is required that these loads are determined accurately with a minimum possible computing time. In the original program the calculations were performed for a fixed number of cycles of iteration. For a given problem size the fixed number of cycles may be too small or too large. If the number of cycles are not enough for accurate calculations of fictitious loads, then the program should be run again with a new higher cycle number and if the number of cycles is too large, then further calculations were deemed unnecessary. The modified program uses only the cycles of iteration needed to achieve the required accuracy. The required accuracy is pre-defined in the input data. The convergence criterion is that no fictitious load at any element may have its magnitude changed by an amount greater than given tolerance limit (ESP) as a result of iteration.

2. Reduced Central Memory: It was necessary to reduce the central storage requirements needed for the original program, because a test example with 110 elements could not be run on the departmental CDC Cyber 730 which allowed 310,000 octal words of central memory. The central storage requirements were reduced by writing all the two-dimensional arrays of the program on two logical units and were recalled whenever needed. This change allowed a larger size problem to be run on the computer. For example, a problem with 110 elements was run with 52,000 octal words of central memory as compared to over 310,000 octal words in the original program.

3. Increased Problem Elements: It was found necessary to increase the element size of the problem when dealing with stress determinations in the neighbourhood of multiple openings. The original program was constructed to deal with up to one hundred elements and any attempt to run the program for more elements produced unacceptable results. The element size of the modified program is limited only by the maximum storage of the computer. To save the unnecessary central storage it is suggested that dimension statements in the program specify only the element size of the problem.

4. Angular Corners: The program provided stress distribution at the grid points on lines starting at the centre of every boundary element. That means that no stress analysis is given in the angular corner regions of excavations. In the modified program stress analysis in the angular corner regions is obtained by forming circular arc corners. An optional program was written to convert the angular corners to circular arc corners. These circular corners were used in the input data for the boundary element program. This optional program was found useful since the corners of most excavations are somewhat rounded.

5. Input/Output: In the modified program the input data is accepted in a free format on logical unit 1. Therefore, it is not necessary to keep the data in any justified form. The output data is also changed for clarity of the printout.

DATA INPUT PROCEDURE

The input data are supplied in the form of card images on a logical unit 1. The input data are grouped into the following categories:

1. Problem title (one card)
2. Problem data (one card)
3. Failure analysis (one card)
4. Field stresses (one card)
5. Boundary segments (one card for each segment)
6. Boundary loads (variable number of cards, omitted if no boundary loads)
7. Program termination (one card).

The preparation of input data is simplified by keeping it in a free format form. Therefore, it is only necessary to separate parameters by one or more blanks (or by a comma or slash). However, more blanks between parameter values can be used to increase its readability by preparing the input data in a tabular form.

During running, a large amount of data is stored on two logical units 1 and 2, and where this information is needed by the program it is automatically taken from these files. After the completion of the program these files are released. The input data preparation should be according to the following instructions:

Card Type		Data Explanation
1.0	TITLE	specify problem title, up to 80 characters only.
2.0	ICODE, NSEG, KXS, KZS, NCYC, EPS, NSL, DELN, E, RNU	
	ICODE	= 1, for infinite medium, = 2, for gravitational medium,
	NSEG	= Total number of segments including straight line, circular arc and elliptical arc segments
	KXS	= 0, for no symmetry about the X-axis = 1, for symmetry about the X-axis The symmetry refers to both the geometry of the boundaries and the disposition of loads. If symmetry option is taken then only one side of the boundaries are to be specified.
	KZS	= 0, for no symmetry about the Z-axis = 1, for symmetry about the Z-axis Other details as for KXS parameter.
	NCYC	= Maximum number of cycles of iteration allowed.
	EPS	= Tolerance used in convergence test, a value of 0.01 is sufficient for most of the problems.
	NSL	= Required number of grid lines parallel to the boundary. The internal stresses are determined and printed at the intersection of these grid lines and the lines drawn normal to the boundaries at the centre of every boundary element. The value of NSL should be large enough to cover the area of interest.
	DELN	= Normal distance (m) between boundary and first grid line. The distance between any two adjacent grid lines is also DELN.
	E	= Young's modulus (MPa) of the rock mass. For typical hard rock mines the value varies from 35 to 105000 MPa.
	RNU	= Poisson's ratio of the rock mass. For typical hard rock mines its value varies from 0.15 to 0.33.
3.0	SIGC, RM, S	
	SIGC	= Uniaxial compressive strength (MPa) of intact rock material.
	RM, S	= Strength parameters which depend upon the properties of the rock and upon the extent to which it has been broken before being subjected to the principal stresses.
		If a failure analysis is not required then the above values should be specified as 0.0.
4.0		This card type defines the field stresses for the medium. For ICODE = 1, Card type 4.1 should be specified and for ICODE = 2, card type 4.2 should be specified.

4.1 FPXX, FPZZ

FPXX = Principal field stress (MPa) in the X-direction
FPZZ = Principal field stress (MPa) in the Z-direction

Where the medium is infinite, FPXX and FPZZ are uniform.

4.2 GAMMA, FSR

GAMMA = Unit weight (MN/m^3) of rock mass
FSR = Field stress ratio, FPXX/FPZZ

If the medium is gravitational, FPXX and FPZZ vary with depth.

5.0 This card type defines the boundary of the excavation. The boundary can be divided into a number of segments consisting of straight line segments, circular arc segments and elliptical arc segments. The initial and final points referenced below follow the rule that when the boundary of the excavation is traced from the initial to the final point and one faces the direction of travel, the solid material lies on the right hand side, e.g., for a circular opening, in an anti-clockwise direction. The following input data is used depending on the segment type, but segment type data should be given in a continuous manner to close an opening. The input format contains some zeros, which should be included in the input data as it is. They are required due to the nature of the free format input. This card type should have a total of NSEG cards.

5.1 Straight line segment (Fig. 1a).

NELR, XO, ZO, XL, ZL, 0.0, 0.0, 0.0

NELR = Number of elements in a segment. Each segment is divided into a number of elements. A large number of elements should be specified in regions of high stress gradient or region of main interest. All elements in a straight line or circular arc segment are of equal length. The length of elements in an elliptical arc varies, being small where the radius of curvature is small. If necessary a single element may be divided into a number of separate elements.
XO,ZO = Coordinates of the initial point
XL,ZL = Coordinates of the final point.

5.2 Circular arc segment (Fig. 1b)

NELR, XC, ZC, THETA1, THETA2, RADIUS, 0.0, 0.0

NELR = Number of elements in circular arc segment
XC,ZC = Coordinates of centre of circle
THETA1, THETA2 = Polar angles in degrees of initial and final point. The polar angles are measured in a counter-clockwise direction from a line, which passes through the centre (XC,ZC) in the positive Z-direction.
RADIUS = Radius of circle.

5.3 Elliptical arc segment (Fig. 1c)
NELR, XC, ZC, THETA1, THETA2, SEMIAX(a), RATIO(b/a), PSI(of a)

NELR = Number of elements in elliptical arc segment
XC,ZC = Coordinates of centre
THETA1, THETA2 = Polar angles in degrees of initial and final point
SEMIAX = Length of one semi-axis(a)
RATIO = b/a, where b is the length of the other semi-axis
PSI = Polar angle of semi-axis(a)

6.0 LP1, LP2, BPX, BPZ

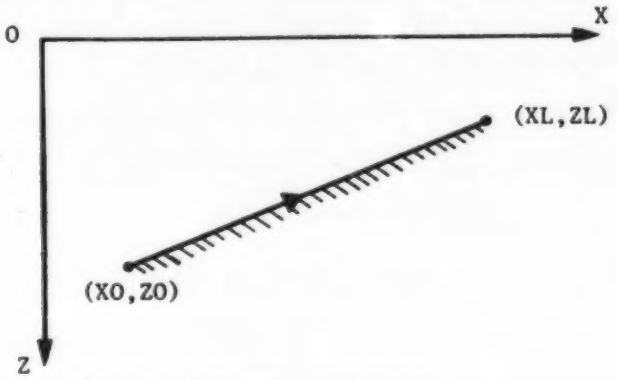
LP1,LP2 = First and last element number of the range of elements over which load is applied. The elements are numbered progressively from 1 to total number of elements
BPX,BPZ = Components of the load applied to a range of boundary elements per unit of area in X and Z directions. BPX and BPZ are positive into the material.

Any number of cards may be specified for this card type. In the absence of boundary loads no card is needed.

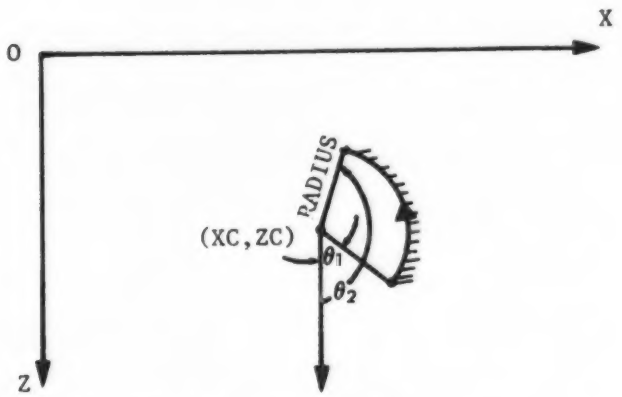
7.0 The last card should contain zero values assigned to parameters in 6.0, i.e.,
0 0 0.0 0.0

PRINTOUT INTERPRETATION

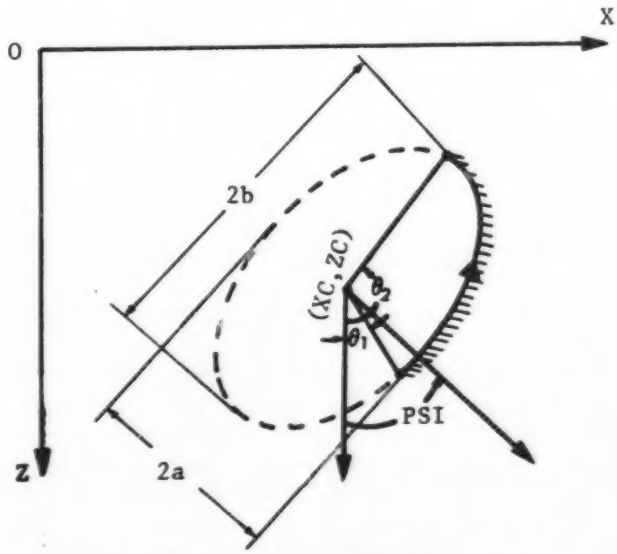
The output is written on a logical unit 7 and printed on a hard copy. The first page of printout contains the input data, date and time the program was executed. The second page contains the number of iteration cycles required in determining fictitious loads. If the maximum number of cycles of iteration specified in the input data is not sufficient to achieve the specified tolerance of convergence test, then a warning is given. In that case the program should be run again with a higher value of the maximum number of iteration cycles (NCYC). However, a following check can also be made to determine whether a sufficient number of cycles of iteration has been achieved. For an element which carries only normal loading (or no load at all), the values of SIG1 and SIG3 at the boundary should be equal to the intensity of loading (or zero) and the ALPHA value should be 0.0 or 90.0 degrees. In other cases the values of SIG1, SIG3 and ALPHA should produce the required intensities



a) Straight line segment.



b) Circular arc segment.



c) Elliptical arc segment.

Fig. 1 - Boundary segments.

of normal and shear loading.

The main printout contains the stress, displacements and failure criterion at the boundary and interior grid points. The meaning of symbols used in the printout are as follows:

NN = 0, precedes the results at centres of boundary elements.

= n, precedes the results at the interior points at the n^{th} line from the boundary.

I = Normal grid line number passing through the centre of I^{th} boundary element (= Element number for boundary stress distribution).

CX,CZ = Coordinates of an interior point (= coordinates of centre of element for boundary stress distribution).

SIG1,SIG3 = Principal stresses.

ALPHA = Angle that SIG1 makes with the Z-axis (= angle that SIG1 makes with the normal to the boundary for boundary stress distribution).

UX,UZ = Displacements in the X and Z direction.

F.O.S = Factor of safety.

BETA = Angle made by the failure plane with the SIG1 direction.

FAILURE = Failure mode (shear, tensile or no failure).

It should be emphasized that because of the method used for calculating displacements, there may be a tendency for the excavation and its surroundings to float numerically in space causing small rigid body movements of the problem field to occur. This can happen for unsymmetrical excavation shapes in which there is an imbalance of elements on opposite sides of the excavation boundary. In these cases, all calculated relative displacements will be accurate, but the absolute values of displacements at particular points may be in error. This problem does not arise for symmetrical excavation shapes.

CIRCULAR ARC SEGMENT PROGRAM

An optional program was written to convert angular corners of polygonal shape excavations to circular arc segments for use in the boundary element program. It was found necessary to approximate the angular corners by more realistic circular corners, which also provided complete stress analyses in the interior of corner regions while using the boundary element program. The complete listing of this program is given in Appendix A.

The input data procedure is explained by using Fig. 2, which shows the geometry of a polygonal shape excavation. The line segments are from (X1,Z1) to (X2,Z2) and (X2,Z2) to (X3,Z3). The line segment directions are according to the same rule as described in card type 5.0. This program converts the angular corner at (X2,Z2) to the circular corner. The parameters for the circular arc segment so obtained are directly used for the boundary element program.

The input data is supplied in the form of card images in a free format on a logical file unit 1. The input data are as follows:

Type Card	Date Explanation
1.0	TITLE specify problem title up to 80 characters.
2.0	This card type requires the coordinates of three points. One card is needed for each angular corner.
	X1, Z1, X2, Z2, X3, Z3, CL (X1,Z1) = Coordinates of initial point of the first line segment. (X2,Z2) = Coordinates of point where both line segments meet. (X3,Z3) = Coordinates of final point on the second line segment. CL = Distance from (X2,Z2) to a point on either line segment where resulting circular arc touches the line segment.

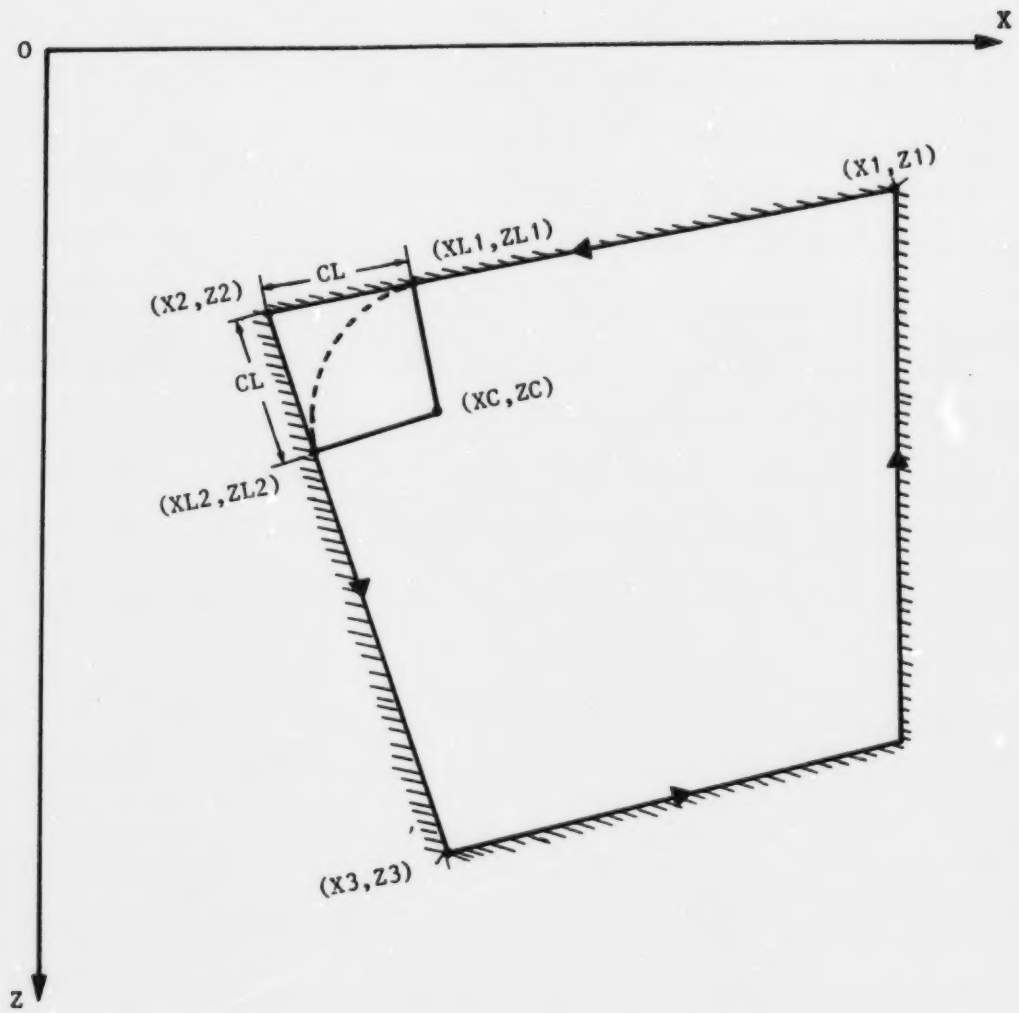


Fig. 2 - Modelling of circular arc segment.

3.0 The last card should contain zero values assigned to parameters in 2.0 card type, i.e.,
0.0 0.0 0.0 0.0 0.0 0.0 0.0

The output is written on logical unit 7 and printed on the hard copy. The meaning of symbols printed are as follows:

(XL1,ZL1) = Coordinates of point on first line segment where the circular arc segment touches it.
(XL2,ZL2) = Coordinates of point on second line segment where the circular arc segment touches it.
(XC,ZC) = Coordinates of centre of circular arc segment.
THETA1,THETA2 = Polar angles in degrees of initial and final point for circular arc segment. The polar angles are measured in a counter-clockwise direction from a line, which passes through the centre (XC,ZC) in the positive Z-direction.
RADIUS = Radius of circular arc.

TEST EXAMPLE

An example is considered to show the running of the boundary element computer program. The program models the stress distribution in the rock surrounding two adjacent excavations inclined at 26.57 degrees at a depth of 515 m below surface as shown in Fig. 3. The following properties of the rock mass are used.

Young's modulus	E = 40,000 MPa
Poisson's ratio	RNU = 0.20
Uniaxial compressive strength of intact rock	SIGC = 160 MPa
Strength parameters	RM = 1.5
	S = 0.004
Unit weight	GAMMA = 0.027 MN/m^3
Horizontal/vertical stress ratio	FSR = 1.5

Before running the boundary element program, it is desired to convert the angular corners of the excavation geometry by circular arcs using the circular arc program. The reason is that in any excavation work the corners are more realistically approximated by circular arcs rather than by angular corners. Also this way the stress distribution will be available at the interior

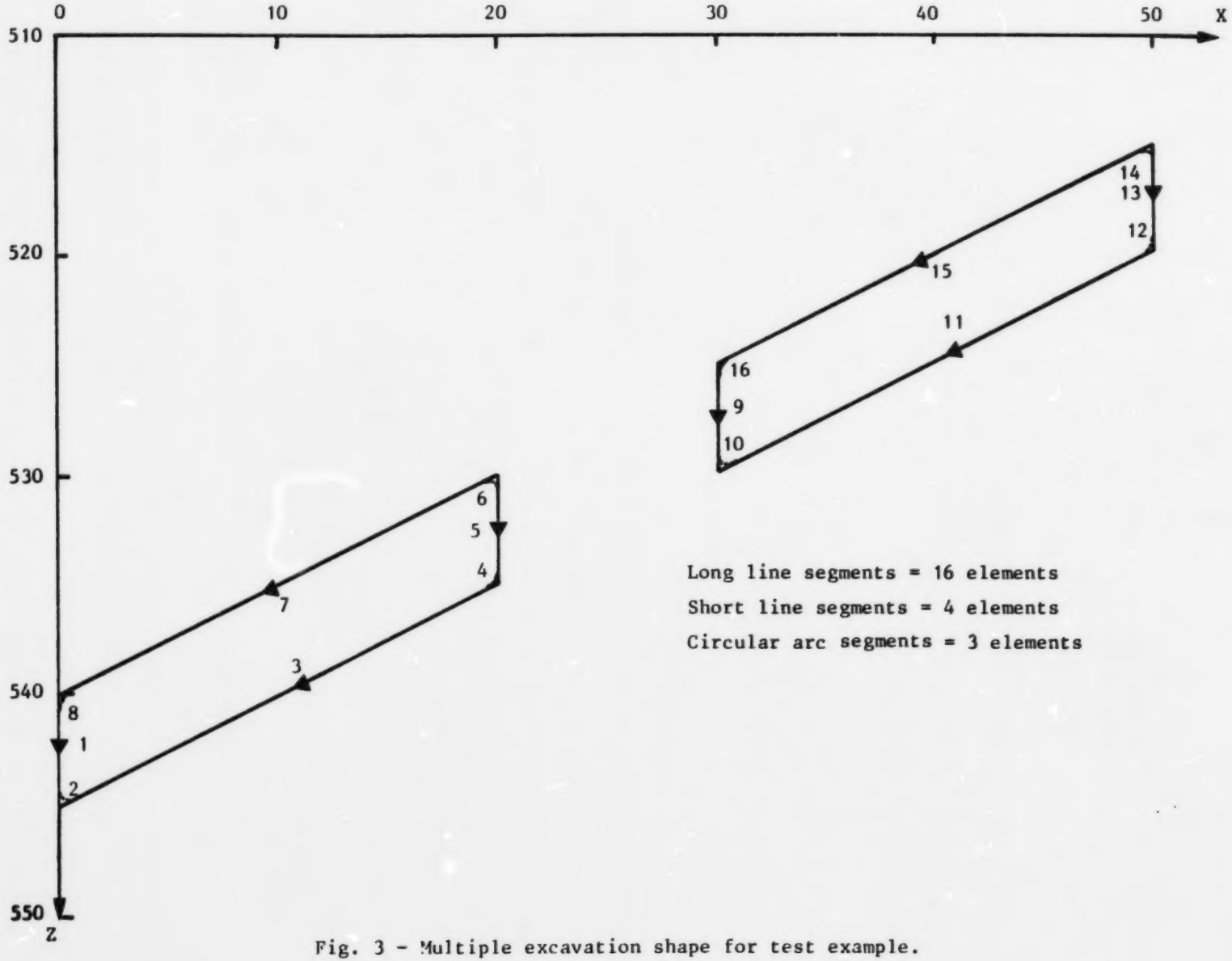


Fig. 3 - Multiple excavation shape for test example.

corner regions while using the boundary elementary program. This is done by using the circular arc segment program. The input data to convert all the angular corners of Fig. 3 to circular arc segments are given in Fig. 4. The complete output of the circular arc program is given in Fig. 5, with all symbols defined earlier. The excavation geometry with angular corners is approximated by the geometry with circular arc corners. This is shown by dotted lines in Fig. 3. This new geometry will be used for the boundary element analysis.

The complete excavation boundary is represented by sixteen segments as shown in Fig. 3. The longer line segments are sub-divided into 16 elements, the shorter line segments into 4 elements and circular arc segments into 3 elements. Therefore, a total of 104 elements is used. The complete input data for boundary element programs are shown in Fig. 6. The coordinates for all initial and final points for all the segments are taken from the output of the circular arc program in Fig. 4. The printing of interior stress analyses is limited by two grid lines with a normal distance between grid lines of 0.40 m. This limited the printout of results. The extended printing of more interior regions could be obtained by increasing the value of the parameter NSL.

The output for the test example is given in Appendix C. The example required a total of 32 iterations. The results clearly show a significant zone of potential failure.

ACKNOWLEDGEMENT

I would like to thank Dr. D.G.F. Hedley for his many suggestions during the development of the model.

```
TEST EXAMPLE FOR TWO INCLINED EXCAVATIONS
20.     530.    0.      540.    0.      545.    0.3
0.      540.    0.      545.    20.     535.    0.3
0.      545.    20.     535.    20.     530.    0.3
20.     535.    20.     530.    0.      540.    0.3
50.     515.    30.     525.    30.     530.    0.3
30.     525.    30.     530.    50.     520.    0.3
30.     530.    50.     520.    50.     515.    0.3
50.     520.    50.     515.    30.     525.    0.3
0.      0.      0.      0.      0.      0.      0.
```

Fig. 4 - Input data to approximate angular corners by circular arc segments.

CIRCULAR ARC SEGMENTS FOR USE IN BOUNDARY ELEMENT PROGRAM 21/10/82 14.47.04.

TEST EXAMPLE FOR TWO INCLINED EXCAVATIONS

N	XL1	ZL1	XL2	ZL2	XC	ZC	THETA1	THETA2	RADIUS
1	.27	539.87	0.00	540.30	.49	540.30	206.57	270.00	.49
2	0.00	544.70	.27	544.87	.19	544.70	270.00	386.57	.19
3	19.73	535.13	20.00	534.70	19.51	534.70	26.57	90.00	.49
4	20.00	530.30	19.73	530.13	19.81	530.30	90.00	206.57	.19
5	30.27	524.87	30.00	525.30	30.49	525.30	206.57	270.00	.49
6	30.00	529.70	30.27	529.87	30.19	529.70	270.00	386.57	.19
7	49.73	520.13	50.00	519.70	49.51	519.70	26.57	90.00	.49
8	50.00	515.30	49.73	515.13	49.81	515.30	90.00	206.57	.19

Fig. 5 - Computer printout for the approximation of angular corners by circular arc segments.

```
STRESS ANALYSIS FOR TWO ADJACENT EXCAVATIONS INCLINED AT 26.57 DEGREES.
2  16  0  0  100  0.01  2  0.4  40000.00  0.200
160.00  7.50  0.10
0.027  1.50
 4      0.00      540.30      0.00      544.70      0.00      0.00      0.00
 3      0.19      544.70    270.00      386.57      0.19      0.00      0.00
16      0.27      544.87     19.73      535.13      0.00      0.00      0.00
 3     19.51      534.70     26.57       97.00      0.49      0.00      0.00
 4     20.00      534.70     90.00      530.30      0.00      0.00      0.00
 3     19.81      530.30     90.00      206.57      0.19      0.00      0.00
16     19.73      530.13      0.27      539.87      0.00      0.00      0.00
 3      0.49      540.30    206.57      270.00      0.49      0.00      0.00
 4     30.00      525.30     30.00      529.70      0.00      0.00      0.00
 3     30.19      529.70    270.00      386.57      0.19      0.00      0.00
16     30.27      529.87     49.73      529.13      0.00      0.00      0.00
 3     49.51      519.70     26.57       99.00      0.49      0.00      0.00
 4     50.00      519.70     90.00      515.30      0.00      0.00      0.00
 3     49.81      515.30     90.00      206.57      0.19      0.00      0.00
16     49.73      515.13     30.27      524.87      0.00      0.00      0.00
 3     30.49      525.30    206.57      270.00      0.49      0.00      0.00
 0      0          0.00       0.00
```

Fig. 6 - Input for boundary element program.

REFERENCES

1. Brady, B.H.G. and Bray, J.W., "The boundary element method for elastic analysis of tabular orebody extraction, assuming complete plane strain"; Int J Rock Mech Min Sci & Geomech, vol 15, pp 29-37; 1978.

2. Brebbia, C.A., "The boundary element method for engineers"; John Wiley and Sons, New York; 1978.

3. Brebbia, C.A. and Walker, S., "Boundary element techniques in engineering"; Newnes-Butterworths; 1980.

4. Hoek, E. and Brown, E.T., "Underground excavation in rock"; published by Inst of Min and Met, London, U.K.; 1980.

APPENDIX A

Circular Arc Segment Program Listing

```
      PROGRAM CIRARC(OUTPUT,TAPE1,TAPE7=OUTPUT)                          A   1
C                                                                        A   2
C     PROGRAM TO JOIN TWO STRAIGHT LINE SEGMENTS BY A CIRCULAR ARC       A   3
C     SEGMENT                                                            A   4
C                                                                        A   5
      DIMENSION TITLE(8)                                                 A   6
      COMMON XC,ZC                                                       A   7
      CALL DATE (DAT)                                                    A   8
      CALL TIME (TIM)                                                    A   9
      WRITE (7,150) DAT,TIM                                              A  10
      READ (1,155) (TITLE(I),I=1,8)                                      A  11
      WRITE (7,160) (TITLE(I),I=1,8)                                     A  12
      WRITE (7,165)                                                      A  13
      N=0                                                                A  14
C                                                                        A  15
C     READING OF INPUT DATA                                              A  16
C                                                                        A  17
100   READ (1,*) X1,Z1,X2,Z2,X3,Z3,CL                                    A  18
      N=N+1                                                              A  19
      IF (CL.EQ.0.0) GO TO 145                                           A  20
C                                                                        A  21
C     CALCULATIONS ON THE FIRST STRAIGHT LINE SEGMENT                    A  22
C                                                                        A  23
      X12=X2-X1                                                          A  24
      Z12=Z2-Z1                                                          A  25
      DIS12=SQRT(X12*X12+Z12*Z12)                                        A  26
      IF (DIS12.EQ.0.0) GO TO 125                                        A  27
      XL1=X1+(DIS12-CL)*X12/DIS12                                        A  28
      ZL1=Z1+(DIS12-CL)*Z12/DIS12                                        A  29
C                                                                        A  30
C     CALCULATIONS ON THE SECOND STRAIGHT LINE SEGMENT                   A  31
C                                                                        A  32
      X23=X3-X2                                                          A  33
      Z23=Z3-Z2                                                          A  34
      DIS23=SQRT(X23*X23+Z23*Z23)                                        A  35
      IF (DIS23.EQ.0.0) GO TO 130                                        A  36
      XL2=X2+CL*X23/DIS23                                                A  37
      ZL2=Z2+CL*Z23/DIS23                                                A  38
C                                                                        A  39
C     DETERMINATION OF THE CENTRE AND RADIUS OF THE CIRCULAR ARC         A  40
C                                                                        A  41
      IF (X23.EQ.0.0.AND.X12.EQ.0.0) GO TO 135                           A  42
      IF (X23.EQ.0.0) GO TO 105                                          A  43
      IF (X12.EQ.0.0) GO TO 110                                          A  44
      SL12=Z12/X12                                                       A  45
      SL23=Z23/X23                                                       A  46
      IF (SL12.EQ.SL23) GO TO 135                                        A  47
      IF (SL12.EQ.0.0) GO TO 115                                         A  48
      IF (SL23.EQ.0.0) GO TO 120                                         A  49
      XC=(SL12*SL23*(ZL2-ZL1)+SL12*XL2-SL23*XL1)/(SL12-SL23)             A  50
```

```
      ZC=ZL1+(XL1/SL12)-(XC/SL12)                                          A  51
      GO TO 140                                                            A  52
  105 SL12=Z12/X12                                                         A  53
      XC=XL1-SL12*(ZL2-ZL1)                                                A  54
      ZC=ZL2                                                               A  55
      GO TO 140                                                            A  56
  110 SL23=Z23/X23                                                         A  57
      XC=XL2+SL23*(ZL2-ZL1)                                                A  58
      ZC=ZL1                                                               A  59
      GO TO 140                                                            A  60
  115 XC=XL1                                                               A  61
      ZC=ZL2-(XC-XL2)/SL23                                                 A  62
      GO TO 140                                                            A  63
  120 XC=XL2                                                               A  64
      ZC=ZL1-(XC-XL1)/SL12                                                 A  65
      GO TO 140                                                            A  66
  125 WRITE (7,170)                                                        A  67
      GO TO 100                                                            A  68
  130 WRITE (7,175)                                                        A  69
      GO TO 100                                                            A  70
  135 WRITE (7,180)                                                        A  71
      GO TO 100                                                            A  72
  140 RDS=SQRT((XC-XL1)**2+(ZC-ZL1)**2)                                    A  73
C                                                                          A  74
C     DETERMINATION OF INITIAL AND FINAL ANGLES FOR THE CIRCULAR ARC       A  75
C                                                                          A  76
      CALL ANGLE (XL1,ZL1,THETA1)                                          A  77
      CALL ANGLE (XL2,ZL2,THETA2)                                          A  78
      IF (THETA2.LT.THETA1) THETA2=360+THETA2                              A  79
      WRITE (7,185) N,XL1,ZL1,XL2,ZL2,XC,ZC,THETA1,THETA2,RDS              A  80
      GO TO 100                                                            A  81
  145 STOP                                                                 A  82
C                                                                          A  83
  150 FORMAT (1H1,///,10X,"CIRCULAR ARC SEGMENTS FOR USE IN BOUNDARY",     A  84
     1  " ELEMENT PROGRAM",2(1X,A10)///)                                  A  85
  155 FORMAT (8A10)                                                        A  86
  160 FORMAT (10X,8A10///)                                                 A  87
  165 FORMAT (10X,1HN,8X,3HXL1,6X,3HZL1,6X,3HXL2,6X,3HZL2,7X,2HXC,         A  88
     1  7X,2HZC,3X,6HTHETA1,3X,6HTHETA2,3X,6HRADIUS/)                     A  89
  170 FORMAT (5X,"WARNING--FIRST TWO POINTS ARE IDENTICAL")                A  90
  175 FORMAT (5X,"WARNING--LAST TWO POINTS ARE IDENTICAL")                 A  91
  180 FORMAT (5X," WARNING--BOTH LINES ARE PARALLEL")                      A  92
  185 FORMAT (9X,I2,2X,9F9.2)                                              A  93
      END                                                                  A  94-
```

```
      SUBROUTINE ANGLE (XL1,ZL1,THETA)                                  B    1
C                                                                       B    2
C     SUBROUTINE TO CALCULATE ANGLES FOR THE CIRCULAR ARC               B    3
C                                                                       B    4
      COMMON XC,ZC                                                      B    5
      PAI=ATAN(1.0)*4.0                                                 B    6
      XAC=XL1-XC                                                        B    7
      ZBC=ZL1-ZC                                                        B    8
      IF (XAC.EQ.0.0.AND.ZBC.EQ.0.0) GO TO 5                            B    9
      IF (XAC.EQ.0.0) SLOPE=PAI/2.0                                     B   10
      IF (XAC.NE.0.0) SLOPE=ATAN(ABS(ZBC/XAC))                          B   11
      IF (XAC.GE.0.0.AND.ZBC.GE.0.0) THETA=90.0-SLOPE*180.0/PAI         B   12
      IF (XAC.GE.0.0.AND.ZBC.LT.0.0) THETA=90.0+SLOPE*180.0/PAI         B   13
      IF (XAC.LT.0.0.AND.ZBC.LT.0.0) THETA=270.0-SLOPE*180.0/PAI        B   14
      IF (XAC.LT.0.0.ANO.ZBC.GE.0.0) THETA=270.0+SLOPE*180.0/PAI        B   15
      GO TO 10                                                          B   16
5     WRITE (7,15)                                                      B   17
10    CONTINUE                                                          B   18
      RETURN                                                            B   19
C                                                                       B   20
15    FORMAT (///,10X,"WARNING-THE CIRCULAR ARC IS OF ZERO RADIUS"//)   B   21
      END                                                               B   22-
```

APPENDIX B

Boundary Element Program Listing

```
      PROGRAM BEP(INPUT,OUTPUT,TAPE2,TAPE3,TAPE1=INPUT,TAPE7=OUTPUT)
C
C     BOUNDARY ELEMENT PROGRAM FOR TWO DIMENSIONAL STRESS DISPLACEMENT
C     AND FAILURE ANALYSIS IN HOMOGENEOUS ISOTROPIC MEDIA UNDER UNIFORM
C     OR GRAVITATIONAL LOADING .
C
      INTEGER FLAG
      DIMENSION CX(110), CZ(110), EX1(110), EX2(110), EZ1(110), EZ2(110)
     1, PN(110), PNM(110), QM(110), QN(110), SB(110), CB(110), SIG1(110)
     2, SIG3(110), BETA(110), FPX(110), FPZ(110), TITLE(8), ALPHA(110),
     3SINB(110), COSB(110), UX(110), UZ(110), FOS(110)
C
C     READING AND PRINTING OF INPUT DATA
C
      CALL DATE (DAT)
      CALL TIME (TIM)
      WRITE (7,330) DAT,TIM
      READ (1,335) (TITLE(I),I=1,8)
      WRITE (7,340) (TITLE(I),I=1,8)
      READ (1,*) ICODE,NSEG,KXS,KZS,NCYC,EPS,NSL,DELN,E,RNU
      WRITE (7,345) ICODE,NSEG,KXS,KZS,NCYC,EPS,NSL,DELN,E,RNU
      IF (ICODE.GE.1.AND.ICODE.LE.2) GO TO 100
      WRITE (7,350)
      GO TO 325
  100 CONTINUE
      READ (1,*) SIGC,RM,S
      IF (SIGC.EQ.0.0) GO TO 105
      WRITE (7,355) SIGC,RM,S
  105 CONTINUE
      IF (ICODE.EQ.2) GO TO 110
      WRITE (7,360)
      READ (1,*) FPXX,FPZZ
      WRITE (7,365) FPXX,FPZZ
      GO TO 115
  110 WRITE (7,370)
      READ (1,*) GAMMA,FSR
      WRITE (7,375) GAMMA,FSR
  115 CONTINUE
      NN=0
      PI=ATAN(1.0)*4.0
      TA=2.0*(1.0-RNU)
      TJ=1.0/(2.0*PI*TA)
      TU=3.0-4.0*RNU
      G=E/(2.0*(1.0+RNU))
      TV=0.5*TJ/G
C
C     INTERPRETATION OF SYMMETRY CODE
C
      KAS=0
      IF (KZS.EQ.-1) KAS = 1
      KXT=2*KXS+1
      KZT=2*(KZS+KAS)+1
C
C     DIVISION OF BOUNDARY INTO ELEMENTS
C
      I=0
      NSEGG=0
```

```
      WRITE (7,380)
C
120   IF (NSEGG.EQ.NSEG) GO TO 150
      NSEGG=NSEGG+1
      NELG=0
      RELG=NELG
      READ (1,*) NELR,XO,ZO,XL,ZL,RDS,RATIO,PSI
      RELR=NELR
      IF (RDS.EQ.0.0) GO TO 140
C
C     DIVISION OF ELLIPTICAL OR CIRCULAR SEGMENTS INTO ELEMENTS
C
      IF (RATIO.EQ.0.0) RATIO = 1.0
      WRITE (7,385) NELR,XO,ZO,XL,ZL,RDS,RATIO,PSI
      SINPSI=SIN(PSI*PI/180.)
      COSPSI=COS(PSI*PI/180.)
      GD=.1E-10
      GA=RATIO*COS((XL-PSI)*PI/180.)
      IF (ABS(GA).LT.GD) GA = GD
      GB=RATIO*COS((ZL-PSI)*PI/180.)
      IF  (ABS(GB).LT.GD) GB = GD
      CHI1=ATAN2(SIN((XL-PSI)*PI/180.),GA)
      CHI2=ATAN2(SIN((ZL-PSI)*PI/180.),GB)
      DCHI=(CHI2-CHI1)/RELR
      IF (ABS(DCHI).LT.GD) GO TO 125
      GC=DCHI/ABS(DCHI)
      GO TO 130
125   GC=-1.0
130   DCHI=DCHI+((ZL-XL)/ABS(ZL-XL)-GC)*PI/RELR
135   I=I+1
      CHI=CHI1+RELG*DCHI
      EX1(I)=RDS*(COS(CHI)*SINPSI+SIN(CHI)*COSPSI*RATIO)+XO
      EZ1(I)=RDS*(COS(CHI)*COSPSI-SIN(CHI)*SINPSI*RATIO)+ZO
      CHI=CHI+DCHI
      EX2(I)=RDS*(COS(CHI)*SINPSI+SIN(CHI)*COSPSI*RATIO)+XO
      EZ2(I)=RDS*(COS(CHI)*COSPSI-SIN(CHI)*SINPSI*RATIO)+ZO
      CX(I)=0.5*(EX1(I)+EX2(I))
      CZ(I)=0.5*(EZ1(I)+EZ2(I))
      DX=EX2(I)-EX1(I)
      DZ=EZ2(I)-EZ1(I)
      IF (ABS(DX).LT.(.1E-13)*RDS) DX = 0.0
      IF  (ABS(DZ).LT.(.1E-13)*RDS) DZ = 0.0
      DS=SQRT(DX*DX+DZ*DZ)
      SINB(I)=-DZ/DS
      COSB(I)=DX/DS
      NELG=NELG+1
      RELG=NELG
      IF (NELG.LT.NELR) GO TO 135
      GO TO 120
C
C     DIVISION OF STRAIGHT LINE SEGMENTS INTO ELEMENTS
C
140   CONTINUE
      WRITE (7,390) NELR,XO,ZO,XL,ZL
      DX=(XL-XO)/RELR
      DZ=(ZL-ZO)/RELR
      DS=SQRT(DX*DX+DZ*DZ)
```

```
      145   I=I+1
            SINB(I)=-DZ/DS
            COSB(I)=DX/DS
            EX1(I)=XO+RELG*DX
            EZ1(I)=ZO+RELG*DZ
            CX(I)=EX1(I)+0.5*DX
            CZ(I)=EZ1(I)+0.5*DZ
            EX2(I)=EX1(I)+DX
            EZ2(I)=EZ1(I)+DZ
            NELG=NELG+1
            RELG=NELG
            IF (NELG.LT.NELR) GO TO 145
            GO TO 120
      150   MAXI=I
            MAXJ=I
      C
      C     DETERMINATION OF BOUNDARY TRACTIONS EQUIVALENT TO FIELD STRESSES
      C
            DO 165 I=1,MAXI
            IF (ICODE.EQ.1) GO TO 155
            FPZ(I)=GAMMA*CZ(I)
            FPX(I)=FSR*FPZ(I)
            GO TO 160
      155   FPZ(I)=FPZZ
            FPX(I)=FPXX
      160   PN(I)=2.0*(FPZ(I)*(COSB(I))**2+FPX(I)*(SINB(I))**2)
            PNM(I)=2.0*(FPX(I)-FPZ(I))*SINB(I)*COSB(I)
            QM(I)=PNM(I)
            QN(I)=PN(I)
      165   CONTINUE
      C
      C     ADDITION OF BOUNDARY TRACTIONS DUE TO BOUNDARY LOADS
      C
      170   READ (1,*) LP1,LP2,BPX,BPZ
            IF (LP1.EQ.0) GO TO 180
            WRITE (7,395) LP1,LP2,BPX,BPZ
            DO 175 I=LP1,LP2
            PN(I)=PN(I)-2.0*((BPX-BPZ)*(SINB(I))**2+BPZ)
            PNM(I)=PNM(I)-2.0*(BPX-BPZ)*SINB(I)*COSB(I)
      175   CONTINUE
            GO TO 170
      C     DETERMINATIONS OF COEFFICIENTS IN EXPRESSIONS FOR STRESSES
      C     INDUCED BY FICTITIOUS LOADS
      C
      180   CONTINUE
            REWIND 2
            REWIND 3
            DO 210 I=1,MAXI
            CXI=CX(I)
            CZI=CZ(I)
            IF (NN.GT.0) GO TO 185
            COS2BI=2.0*(COSB(I))**2-1.0
            SIN2BI=2.0*SINB(I)*COSB(I)
      185   DO 205 J=1,MAXJ
            TK=0.0
            TL=0.0
            TM=0.0
```

```
                TN=0.0
                TO=0.0
                TP=0.0
175             DO 200 KXU=1,KXT,2
                KX=2-KXU
                RX=KX
                DO 200 KZU=1,KZT,2
                KZ=(2-KZU)*(1-KAS)+KAS*KX
180             RZ=KZ
                COSBJ=RX*COSB(J)
                SINBJ=RZ*SINB(J)
                EX1J=RZ*EX1(J)
                EX2J=RZ*EX2(J)
185             E71J=RX*EZ1(J)
                EZ2J=RX*EZ2(J)
                KXZ2=KX+KZ-2
                IF (NN.EQ.0.AND.I.EQ.J.AND.KXZ2.EQ.0) GO TO 190
                RN=(CZI-EZ1J)*COSBJ+(CXI-EX1J)*SINBJ
190             RM1=(CXI-EX1J)*COSBJ-(CZI-EZ1J)*SINBJ
                RM2=(CXI-EX2J)*COSBJ-(CZI-EZ2J)*SINBJ
                RSQ1=RM1*RM1+RN*RN
                RSQ2=RM2*RM2+RN*RN
                TB=2.0*ATAN2((RM1-RM2)*RN,(RN*RN+RM1*RM2))
195             TC=2.0*RN*(RM1/RSQ1-RM2/RSQ2)
                TD=(RN**2-RM1**2)/RSQ1-(RN**2-RM2**2)/RSQ2
                TE=ALOG(RSQ1/RSQ2)
                COS2F=2.0*(COS2BI*(COSBJ**2-0.5)+SIN2BI*SINBJ*COSBJ)
                SIN2F=2.0*(SIN2BI*(COSBJ**2-0.5)-COS2BI*SINBJ*COSBJ)
200             GO TO 195
            190 TA=2.0*PI
                TC=0.0
                TD=0.0
                TE=0.0
205             COS2F=1.0
                SIN2F=0.0
            195 CONTINUE
                TK=TK+TB*RX*RZ
                TL=TL+TE
210             TM=TM+(TD+TA*TE)*COS2F+(TC-TA*TB)*SIN2F
                TN=TN+(((1.0-TA)*TB-TC)*COS2F+(TD+(1.0-TA)*TE)*SIN2F)*RX*RZ
                TO=TO+(TD+TA*TE)*SIN2F-(TC-TA*TB)*COS2F
                TP=TP+(((1.0-TA)*TB-TC)*SIN2F-(TD+(1.0-TA)*TE)*COS2F)*RX*RZ
            200 CONTINUE
215             BMM=(TL+TM)*TJ
                BMN=(TK+TN)*TJ
                BNM=(TL-TM)*TJ
                BNN=(TK-TN)*TJ
                DM=TO*TJ
220             DN=TP*TJ
                IF (I.NE.J) WRITE (2) BMM,BMN,BNM,BNN,DM,DN
                IF (I.EQ.J) WRITE (3) BMM,BMN,BNM,BNN,DM,DN
            205 CONTINUE
            210 CONTINUE
225             IF (NN.GT.0) GO TO 230
            C
            C       DETERMINATION OF FICTITIOUS LOADS
            C
```

```
      DO 225 ITER=1,NCYC
230   REWIND 2
      REWIND 3
      FLAG=1
      DO 220 I=1,MAXI
      QMSTART=QM(I)
235   QNSTART=QN(I)
      QMI=PMM(I)
      QNI=PN(I)
      DO 215 J=1,MAXJ
      IF (I.EQ.J) GO TO 215
240   READ (2) BMM,BMN,BNM,BNN,DM,DN
      QMI=QMI-DM*QM(J)-DN*QN(J)
      QNI=QNI-BNM*QM(J)-BNN*QN(J)
215   CONTINUE
      READ (3) BMM,BMN,BNM,BNN,DM,DN
245   DENOM=BNN*DM-DN*BNM
      QM(I)=(QMI*BNN-QNI*DN)/DENOM
      QN(I)=(QNI*DM-QMI*BNM)/DENOM
      QMCONV=ABS(QMSTART-QM(I))
      QNCONV=ABS(QNSTART-QN(I))
250   IF (QMCONV.LE.EPS.AND.QNCONV.LE.EPS) GO TO 220
      FLAG=0
220   CONTINUE
      IF (FLAG.NE.1) GO TO 225
      IITER=ITER-1
255   WRITE (7,400) IITER
      GO TO 230
225   CONTINUE
      WRITE (7,405) ITER
C
260   C     DETERMINATION OF STRESS COMPONENTS, PRINCIPAL STRESSES
C      AND DIRECTIONS
C
230   CONTINUE
      REWIND 2
265   REWIND 3
      DO 245 I=1,MAXI
      SMI=2.0*FPX(I)
      SNI=2.0*FPZ(I)
      SNMI=0.0
270   IF (NN.GT.0) GO TO 235
      SMI=2.0*((FPX(I)-FPZ(I))*(COSB(I))**2+FPZ(I))
      SNI=2.0*((FPX(I)-FPZ(I))*(SINB(I))**2+FPZ(I))
      SNMI=2.0*(FPX(I)-FPZ(I))*SINB(I)*COSB(I)
235   DO 240 J=1,MAXJ
275   IF (I.EQ.J) READ (3) BMM,BMN,BNM,BNN,DM,DN
      IF (I.NE.J) READ (2) BMM,BMN,BNM,BNN,DM,DN
      SNI=SNI-BNM*QM(J)-BNN*QN(J)
      SMI=SMI-BMM*QM(J)-BMN*QN(J)
      SNMI=SNMI-DM*QM(J)-DN*QN(J)
280   240 CONTINUE
      SDI=0.5*(SMI-SNI)
      TAUMAX=0.5*SQRT(SDI**2+SNMI**2)
      SIG1(I)=0.25*(SMI+SNI)+TAUMAX
      SIG3(I)=0.25*(SMI+SNI)-TAUMAX
285   TR=2.0*TAUMAX+SNMI-SDI
```

```
      IF (TP.EQ.0.0) TR = 0.00001
      ALPHA(I)=(180./PI)*ATAN(1.0+2.*SDI/TR)
  245 CONTINUE
C
C     DETERMINATION OF INDUCED DISPLACEMENTS
C
      DO 260 I=1,MAXI
      CXI=CX(I)
      CZI=CZ(I)
      UXX=0.0
      UZZ=0.0
      DO 255 J=1,MAXJ
      DO 250 KXU=1,KXT,2
      KX=2-KXU
      RX=KX
      DO 250 KZU=1,KZT,2
      KZ=(2-KZU)*(1-KAS)+KAS*KX
      RZ=KZ
      COSBJ=RX*COSB(J)
      SINBJ=RZ*SINB(J)
      EX1J=RZ*EX1(J)
      EX2J=RZ*EX2(J)
      EZ1J=RX*EZ1(J)
      EZ2J=RX*EZ2(J)
      RN=(CZI-EZ1J)*COSBJ+(CXI-EX1J)*SINBJ
      KXZ2=KX+KZ-2
      IF (NN.EQ.0.AND.I.EQ.J.AND.KXZ2.EQ.0) RN=0.0001
      RM1=(CXI-EX1J)*COSBJ-(CZI-EZ1J)*SINBJ
      RM2=(CXI-EX2J)*COSBJ-(CZI-EZ2J)*SINBJ
      RSQ1=SQRT(RM1*RM1+RN*RN)
      RSQ2=SQRT(RM2*RM2+RN*RN)
      RN0=-EZ1J*COSBJ-EX1J*SINBJ
      IF (ABS(RN0).LT.0.0001) RN0 = 0.0001
      RM10=EX1J*COSBJ-EZ1J*SINBJ
      RM20=EX2J*COSBJ-EZ2J*SINBJ
      RSQ10=SQRT(RM10*RM10+RN0*RN0)
      RSQ20=SQRT(RM20*RM20+RN0*RN0)
      IF (RSQ10.LT.0.001) RSQ10 = 0.001
      IF (RSQ20.LT.0.001) RSQ20 = 0.001
      IF (ABS(RN0).LT.0.0001.AND .RSQ20.LT.0.001) RSQ20 = 0.001
      TS=-RN*ALOG(RSQ2/RSQ1)
      TR=-RN*ATAN2((RM2-RM1)*RN,(RN*RN+RM1*RM2))
      TT=TU*(-RM2*ALOG(RSQ2)+RM1*ALOG(RSQ1)+RM2-RM1*TR)
      UM=TV*(TS*QN(J)*RX*RZ-(TT+TR)*QM(J))
      UN=TV*(TS*QM(J)-(TT-RM2+RM1-TR)*RX*RZ*QN(J))
      UXX=UXX-UM*COSBJ-UN*SINBJ
      UZZ=UZZ+UM*SINBJ-UN*COSBJ
  250 CONTINUE
  255 CONTINUE
      UX(I)=UXX*1000.0
      UZ(I)=UZZ*1000.0
  260 CONTINUE
      WRITE (7,410) NN
      IF (SIGC.EQ.0.0) GO TO 290
      IF  (NN.GT.0) GO TO 265
      WRITE (7,415)
      GO TO 270
```

```
  265   WRITE (7,420)
  270   WRITE (7,425)
345     C
        C     FAILURE CRITERION AND FACTOR OF SAFETY DETERMINATIONS
        C
              DO 285 I=1,MAXI
              IF (ABS(SIG1(I)).LT.0.0001) SIG1(I)=0.0001
350           SIGT=0.5*SIGC*(RM-SQRT(RM*RM+4.0*S))
              IF (SIG3(I).LE.SIGT) GO TO 275
        C
        C     CHECK FOR SHEAR FAILURE
        C
355           TSC=SQRT(RM*SIGC*SIG3(I)+S*SIGC*SIGC)
              FOS(I)=(SIG3(I)+TSC)/SIG1(I)
              IF (FOS(I).GT.1.0) GO TO 280
              TAUM=0.5*(SIG1(I)-SIG3(I))
              TCM=0.25*SIGC*RM
360           TCM1=SQRT(TAUM*TAUM+TAUM*TCM)
              TCM2=0.5*TCM
              BETA(I)=0.5*(180.0/PI)*ATAN(TCM1/TCM2)
              WRITE (7,430) (I,CX(I),CZ(I),SIG1(I),SIG3(I),ALPHA(I),UX(I),UZ(I),
             1FOS(I),BETA(I))
365           GO TO 285
        C
        C     CHECK FOR TENSILE FAILURE
        C
        275   TCT=RM-SQRT(RM*RM+4.0*S)
370           FOS(I)=SIGC*TCT/(2.0*SIG3(I))
              IF (FOS(I).GT.1.0) GO TO 280
              BETA(I)=0.0
              WRITE (7,435) (I,CX(I),CZ(I),SIG1(I),SIG3(I),ALPHA(I),UX(I),UZ(I),
             1FOS(I),BETA(I))
375           GO TO 285
        280   WRITE (7,440) (I,CX(I),CZ(I),SIG1(I),SIG3(I),ALPHA(I),UX(I),UZ(I),
             1FOS(I))
        285   CONTINUE
              GO TO 305
380     290   IF (NN.GT.0) GO TO 295
              WRITE (7,445)
              GO TO 300
        295   WRITE (7,450)
        300   WRITE (7,455)
385           WRITE (7,460) (I,CX(I),CZ(I),SIG1(I),SIG3(I),ALPHA(I),UX(I),UZ(I),
             1I=1,MAXI)
        305   CONTINUE
              IF (NN.EQ.NSL) GO TO 325
        C
390     C     GENERATION OF INTERIOR POINTS, STRESSES AND DISPLACEMENTS
        C     DETERMINED BY LOOPING TO 105
        C
              NN=NN+1
              IF (NN.GT.1) GO TO 315
395           COS2BI=1.0
              SIN2BI=0.0
              MAXI=MAXJ
              DO 310 I=1,MAXI
              K=I
```

```
400               CX(I)=CX(K)
                  CZ(I)=CZ(K)
                  SB(I)=SINB(K)
                  CB(I)=COSB(K)
              310 CONTINUE
405           315 TQ=DELN
                  DO 320 I=1,MAXI
                  CX(I)=CX(I)+SB(I)*TQ
                  CZ(I)=CZ(I)+CB(I)*TQ
              320 CONTINUE
410               GO TO 180
              325 CONTINUE
                  STOP
                  C
                  330 FORMAT ("1",///,10X,"2-D STRESS ANALYSIS BY THE BOUNDARY ELEMENT "
415               1, "METHOD ",2(3X,A10),///)
                  335 FORMAT (8A10)
                  340 FORMAT (10X,"INPUT DATA"/,10X,"----------"///,10X,7HTITLE--,8A10)
                  345 FORMAT (1H /,10X,"ICODE,NSEG,KXS,KZS,NCYC,EPS = ",5I5,F10.5/,
                  1  10X,"NSL,DELN,E,RNU = ",I3,3F10.2/)
420               350 FORMAT (1H ///,7X,"CODE OF PROBLEM TYPE IS WRONG")
                  355 FORMAT (1H ,9X,"SIGC, RM, S = ",3F10.3)
                  360 FORMAT (1H /,10X,"CODE OF ANALYSIS IN INFINITE ISOTROPIC MEDIA")
                  365 FORMAT (10X,"FPX, FPZ =",2F10.3)
                  370 FORMAT (1H /,10X,"CODE OF THE ANALYSIS IN HOMOGENEOUS "
425               1  ," GRAVITATIONAL MEDIA  ")
                  375 FORMAT (1H /,10X,"GAMMA, HORZ, VTR RATIO = ",2F10.3)
                  380 FORMAT (///,5X,
                  1   "     ELEMENTS CENT X    CENT Z      THETA1     THETA2",
                  2   "     RADIUS    RATIO     PSI"/
                  3  ,5X,"                (FIRST-X) (FIRST-Z)  (LAST-X)  (LAST-Z)"/)
430               385 FORMAT (1H ,11X,I3,7F10.2)
                  390 FORMAT (1H ,11X,I3,4F10.2)
                  395 FORMAT (/,10X,"LP1 =",I6/10X,"LP2 =",I6/10X,
                  1                 "BPX =",F10.3/10X,"BPZ =",F10.3///)
435               400 FORMAT ("1",10X,"R E S U L T S",/11X,"-------------"///,
                  1  30X,"NUMBER OF ITERATION CYCLES REQUIRED =",I3//)
                  405 FORMAT ("1",10X,"R E S U L T S",/11X,"-------------"///,
                  1  20X,"WARNING--REQUIRED CONVERGENCE NOT ACHIEVED",
                  2  ," WITH ITERATION CYCLES =",I3/20X,"-----------------",/
440               3  20X,"-----------------",///)
                  410 FORMAT ("1",5X,"NN =",I3)
                  415 FORMAT (1H //,4X,"STRESSES AND DISPLACEMENTS AND FAILURE "
                  1  ,"CRITERION AT CENTRES OF BOUNDARY ELEMENTS")
                  420 FORMAT (1H //,4X,"STRESSES AND DISPLACEMENTS AND FAILURE "
445               1    ,"CRITERION AT INTERIOR POINTS")
                  425 FORMAT (1H //,4X,2H I,5X,2HCX,6X,2HCZ,4X,4HSIG1,4X,4HSIG3,4X,
                  1  5HALPHA,4X,3H UX,5X,3H UZ,4X,5HF.O.S,4X,4HBETA,3X,7HFAILURE/,
                  2  1H ,10X,3H(M),5X,3H(M),2X,6H(MAPA),2X,6H(MAPA),3X,5H(DEG),4X,
                  3  4H(MM),4X,4H(MM),11X,5H(DEG)/)
450               430 FORMAT (1H ,I5,2F8.1,2F8.2,F8.1,2F8.3,2F8.1,3X,5HSHEAR)
                  435 FORMAT (1H ,I5,2F8.1,2F8.2,F8.1,2F8.3,2F8.1,2X,7HTENSILE)
                  440 FORMAT (1H ,I5,2F8.1,2F8.2,F8.1,2F8.3,F8.1,5X,3HN.A,4X,2HNO)
                  445 FORMAT (1H //,4X,"STRESSES AND DISPLACEMENTS AT CENTRES OF",
                  1  " BOUNDARY ELEMENTS")
455               450 FORMAT (1H //,4X,"STRESSES AND DISPLACEMENTS AT INTERIOR POINTS")
                  455 FORMAT (1H //,4X,2H I,5X,2HCX,6X,2HCZ,4X,4HSIG1,4X,4HSIG3,4X,
```

```
     1  5HALPHA,4X,3H  UX,5X,3H UZ/,1H ,10X,3H(M),5X,3H(M),2X,
     2  6H(MAPA),2X,6H(MAPA),3X,5H(DEG),4X,4H(MM),4X,4H(MM)/)
460   FORMAT (1H ,I5,2F8.1,2F8.2,F8.1,2F8.3)
460   END
```

APPENDIX C

Multiple Excavation Test Example Printout

2-D STRESS ANALYSIS BY THE BOUNDARY ELEMENT METHOD 05/11/82 17.01.22.

INPUT DATA

TITLE--STRESS ANALYSIS FOR TWO ADJACENT EXCAVATIONS INCLINED AT 26.57 DEGREE.

ICODE,NSEG,KXS,KZS,NCYC,EPS = 2 16 0 0 100 .01000
NSL,DELN,E,RNU = 2 .40 40000.00 .20

SIGC, RM, S = 160.000 1.500 .004

CODE OF THE ANALYSIS IN HOMOGENEOUS GRAVITATIONAL MEDIA

GAMMA, HORZ, VTR RATIO = .027 1.500

ELEMENTS	CENT X (FIRST-X)	CENT Z (FIRST-Z)	THETA1 (LAST-X)	THETA2 (LAST-Z)	RADIUS	RATIO	PSI
4	0.00	540.30	0.00	544.70			
3	.19	544.70	270.00	386.57	.19	1.00	0.00
16	.27	544.87	19.73	535.13			
3	19.51	534.70	26.57	90.00	.49	1.00	0.00
4	20.00	534.70	20.00	530.30			
3	19.81	530.30	90.00	206.57	.19	1.00	0.00
16	19.73	530.13	.27	539.87			
3	.49	540.30	206.57	270.00	.49	1.00	0.00
4	30.00	525.30	30.00	529.70			
3	30.19	529.70	270.00	386.57	.19	1.00	0.00
16	30.27	529.87	49.73	520.13			
3	49.51	519.70	26.57	90.00	.49	1.00	0.00
4	50.00	519.70	50.00	515.30			
3	49.81	515.30	90.00	206.57	.19	1.00	0.00
16	49.73	515.13	30.27	524.87			
3	30.49	525.30	206.57	270.00	.49	1.00	0.00

RESULTS

NUMBER OF ITERATION CYCLES REQUIRED = 32

NN = 0

STRESSES AND DISPLACEMENTS AND FAILURE CRITERION AT CENTRES OF BOUNDARY ELEMENTS

I	CX (M)	CZ (M)	SIG1 (MAPA)	SIG3 (MAPA)	ALPHA (DEG)	UX (MM)	UZ (MM)	F.O.S	BETA (DEG)	FAILURE
1	0.0	540.9	28.97	-.00	-90.0	4.474	5.311	.3	23.8	SHEAR
2	0.0	542.0	23.64	-.00	-90.0	3.745	5.000	.4	22.1	SHEAR
3	0.0	543.1	27.27	-.00	-90.0	2.630	4.726	.4	23.3	SHEAR
4	0.0	544.2	53.01	-.00	-90.0	.766	4.239	.2	29.0	SHEAR
5	.0	544.8	191.84	-.00	-90.0	-1.248	3.263	.1	38.1	SHEAR
6	.1	544.9	305.40	-.00	90.0	-2.012	2.777	.0	40.3	SHEAR
7	.2	544.9	204.47	.00	90.0	-2.804	2.329	.0	38.4	SHEAR
8	.9	544.6	58.57	.00	-90.0	-4.878	.738	.2	29.8	SHEAR
9	2.1	544.0	27.39	.00	-90.0	-6.281	-1.182	.4	23.3	SHEAR
10	3.3	543.3	19.45	.00	-90.0	-7.011	-2.522	.5	20.5	SHEAR
11	4.5	542.7	15.69	.00	-90.0	-7.479	-3.559	.6	18.8	SHEAR
12	5.7	542.1	13.60	.00	-90.0	-7.791	-4.391	.7	17.7	SHEAR
13	7.0	541.5	12.31	.00	-90.0	-7.988	-5.066	.8	17.0	SHEAR
14	8.2	540.9	11.42	.00	-90.0	-8.093	-5.611	.9	16.4	SHEAR
15	9.4	540.3	10.76	.00	-90.0	-8.118	-6.043	.9	16.0	SHEAR
16	10.6	539.7	10.25	.00	-90.0	-8.072	-6.371	1.0	15.7	SHEAR
17	11.8	539.1	9.85	.00	90.0	-7.959	-6.596	1.0	N.A	NO
18	13.0	538.5	9.58	.00	90.0	-7.782	-6.719	1.1	N.A	NO
19	14.3	537.9	9.49	.00	90.0	-7.541	-6.734	1.1	N.A	NO
20	15.5	537.3	9.70	.00	90.0	-7.237	-6.626	1.0	N.A	NO
21	16.7	536.7	10.46	.00	90.0	-6.869	-6.376	1.0	15.8	SHEAR
22	17.9	536.0	12.41	.00	90.0	-6.444	-5.941	.8	17.0	SHEAR
23	19.1	535.4	17.80	.00	90.0	-6.004	-5.262	.6	19.8	SHEAR
24	19.8	535.1	25.63	.00	-90.0	-5.753	-4.713	.4	22.8	SHEAR
25	19.9	535.0	40.17	.00	-90.0	-5.684	-4.493	.3	26.6	SHEAR
26	20.0	534.8	29.86	.00	-90.0	-5.649	-4.334	.3	24.1	SHEAR
27	20.0	534.2	26.77	-.00	-90.0	-5.283	-4.092	.4	23.1	SHEAR
28	20.0	533.1	22.07	-.00	-90.0	-4.363	-3.849	.5	21.5	SHEAR
29	20.0	532.0	27.85	-.00	-90.0	-2.935	-3.612	.4	23.5	SHEAR
30	20.0	530.8	59.67	-.00	-90.0	-.619	-3.072	.2	30.0	SHEAR
31	20.0	530.2	220.93	-.00	-90.0	1.756	-1.927	.0	38.8	SHEAR
32	19.9	530.1	341.16	-.00	90.0	2.620	-1.363	.0	40.7	SHEAR
33	19.8	530.1	219.08	.00	90.0	3.489	-.851	.0	38.8	SHEAR
34	19.1	530.4	57.84	.00	90.0	5.681	.907	.2	29.7	SHEAR
35	17.9	531.0	24.19	.00	90.0	7.034	2.951	.4	22.3	SHEAR
36	16.7	531.7	15.99	.00	90.0	7.653	4.328	.6	18.9	SHEAR
37	15.5	532.3	12.28	.00	90.0	7.996	5.364	.8	16.9	SHEAR
38	14.3	532.9	10.33	.00	90.0	8.179	6.174	1.0	15.7	SHEAR
39	13.0	533.5	9.22	.00	90.0	8.248	6.816	1.1	N.A	NO
40	11.8	534.1	8.50	.00	90.0	8.226	7.321	1.2	N.A	NO
41	10.6	534.7	8.00	.00	90.0	8.128	7.708	1.3	N.A	NO
42	9.4	535.3	7.63	.00	90.0	7.961	7.988	1.3	N.A	NO
43	8.2	535.9	7.37	.00	90.0	7.731	8.165	1.4	N.A	NO
44	7.0	536.5	7.24	.00	90.0	7.440	8.239	1.4	N.A	NO
45	5.7	537.1	7.29	.00	90.0	7.090	8.205	1.4	N.A	NO
46	4.5	537.7	7.63	.00	90.0	6.680	8.053	1.3	N.A	NO
47	3.3	538.3	8.54	.00	90.0	6.213	7.761	1.2	N.A	NO
48	2.1	539.0	10.70	.00	90.0	5.696	7.291	.9	16.0	SHEAR

49	.9	539.6	16.58	.00	90.0	5.174	6.580	.6	19.2	SHEAR
50	.2	539.9	25.85	.00	90.0	4.887	6.004	.4	22.8	SHEAR
51	.1	540.0	42.45	.00	-90.0	4.815	5.776	.2	27.1	SHEAR
52	.0	540.2	33.99	-.00	-90.0	4.781	5.601	.3	25.2	SHEAR
53	30.0	525.9	26.15	-.00	90.0	5.253	4.174	.4	22.9	SHEAR
54	30.0	527.0	21.56	-.00	90.0	4.351	3.940	.5	21.3	SHEAR
55	30.0	528.1	27.33	-.00	90.0	2.941	3.713	.4	23.3	SHEAR
56	30.0	529.2	58.95	-.00	90.0	.645	3.186	.2	29.9	SHEAR
57	30.0	529.8	218.91	-.00	90.0	-1.712	2.054	.0	38.8	SHEAR
58	30.1	529.9	338.42	-.00	90.0	-2.570	1.496	.0	40.7	SHEAR
59	30.2	529.9	217.48	.00	90.0	-3.432	.990	.0	38.8	SHEAR
60	30.9	529.6	57.40	.00	-90.0	-5.603	-.750	.2	29.6	SHEAR
61	32.1	529.0	23.93	.00	-90.0	-6.937	-2.768	.4	22.2	SHEAR
62	33.3	528.3	15.77	.00	-90.0	-7.540	-4.123	.6	18.8	SHEAR
63	34.5	527.7	12.06	.00	-90.0	-7.869	-5.138	.8	16.8	SHEAR
64	35.7	527.1	10.11	.00	-90.0	-8.039	-5.928	1.0	N.A	NO
65	37.0	526.5	8.98	.00	-90.0	-8.097	-6.551	1.1	N.A	NO
66	38.2	525.9	8.25	.00	-90.0	-8.066	-7.038	1.2	N.A	NO
67	39.4	525.3	7.74	.00	-90.0	-7.959	-7.408	1.3	N.A	NO
68	40.6	524.7	7.36	.00	-90.0	-7.785	-7.672	1.4	N.A	NO
69	41.8	524.1	7.09	.00	-90.0	-7.550	-7.834	1.4	N.A	NO
70	43.0	523.5	6.94	.00	-90.0	-7.256	-7.896	1.5	N.A	NO
71	44.3	522.9	6.97	.00	-90.0	-6.904	-7.853	1.5	N.A	NO
72	45.5	522.3	7.29	.00	-90.0	-6.495	-7.693	1.4	N.A	NO
73	46.7	521.7	8.15	.00	-90.0	-6.031	-7.398	1.2	N.A	NO
74	47.9	521.0	10.21	.00	-90.0	-5.519	-6.930	1.0	15.6	SHEAR
75	49.1	520.4	15.86	.00	-90.0	-5.002	-6.230	.6	18.9	SHEAR
76	49.8	520.1	24.72	.00	-90.0	-4.718	-5.666	.4	22.5	SHEAR
77	49.9	520.0	40.87	.00	-90.0	-4.645	-5.443	.2	26.7	SHEAR
78	50.0	519.8	32.86	-.00	-90.0	-4.609	-5.273	.3	24.9	SHEAR
79	50.0	519.2	28.09	-.00	-90.0	-4.301	-4.991	.4	23.5	SHEAR
80	50.0	518.1	22.96	-.00	-90.0	-3.580	-4.685	.4	21.8	SHEAR
81	50.0	517.0	26.47	-.00	-90.0	-2.490	-4.414	.4	23.0	SHEAR
82	50.0	515.9	51.26	-.00	-90.0	-.683	-3.935	.2	28.7	SHEAR
83	50.0	515.2	185.05	-.00	-90.0	1.259	-2.989	.1	37.9	SHEAR
84	49.9	515.1	294.01	-.00	90.0	1.995	-2.520	.0	40.1	SHEAR
85	49.8	515.1	196.52	.00	90.0	2.758	-2.087	.1	38.2	SHEAR
86	49.1	515.4	56.21	.00	90.0	4.755	-.556	.2	29.5	SHEAR
87	47.9	516.0	26.30	.00	90.0	6.113	1.296	.4	23.0	SHEAR
88	46.7	516.7	18.71	.00	90.0	6.825	2.591	.5	20.2	SHEAR
89	45.5	517.3	15.12	.00	90.0	7.286	3.595	.7	18.5	SHEAR
90	44.3	517.9	13.14	.00	90.0	7.596	4.403	.8	17.4	SHEAR
91	43.0	518.5	11.92	.00	90.0	7.796	5.061	.8	16.7	SHEAR
92	41.8	519.1	11.09	.00	90.0	7.908	5.596	.9	16.2	SHEAR
93	40.6	519.7	10.47	.00	90.0	7.942	6.021	1.0	15.8	SHEAR
94	39.4	520.3	9.99	.00	90.0	7.907	6.346	1.0	N.A	NO
95	38.2	520.9	9.63	.00	90.0	7.808	6.573	1.1	N.A	NO
96	37.0	521.5	9.39	.00	90.0	7.645	6.700	1.1	N.A	NO
97	35.7	522.1	9.32	.00	90.0	7.420	6.721	1.1	N.A	NO
98	34.5	522.7	9.55	.00	90.0	7.132	6.624	1.1	N.A	NO
99	33.3	523.3	10.31	.00	90.0	6.781	6.387	1.0	15.7	SHEAR
100	32.1	524.0	12.24	.00	90.0	6.374	5.969	.8	16.9	SHEAR
101	30.9	524.6	17.54	.00	-90.0	5.952	5.311	.6	19.7	SHEAR
102	30.2	524.9	25.19	.00	-90.0	5.710	4.778	.4	22.6	SHEAR
103	30.1	525.0	39.35	-.00	-90.0	5.644	4.563	.3	26.4	SHEAR
104	30.0	525.2	29.10	.00	90.0	5.610	4.408	.3	23.8	SHEAR

NN = 1

STRESSES AND DISPLACEMENTS AND FAILURE CRITERION AT INTERIOR POINTS

I	CX (M)	CZ (M)	SIG1 (MAPA)	SIG3 (MAPA)	ALPHA (DEG)	UX (MM)	UZ (MM)	F.O.S	BETA (DEG)	FAILURE
1	-.4	540.9	24.72	2.23	-6.6	4.252	5.018	1.1	N.A	NO
2	-.4	542.0	23.96	1.70	-1.2	3.525	4.645	1.0	N.A	NO
3	-.4	543.1	29.51	2.09	4.7	2.403	4.211	.9	23.3	SHEAR
4	-.4	544.2	58.31	5.29	20.6	.518	3.457	.7	29.0	SHEAR
5	-.4	544.9	89.76	33.50	57.4	-1.382	2.846	1.4	N.A	NO
6	-.1	545.2	83.59	55.85	63.0	-2.013	2.692	2.1	N.A	NO
7	.3	545.3	90.74	36.91	62.1	-2.411	2.199	1.5	N.A	NO
8	1.1	544.9	59.73	5.08	-81.8	-4.004	.515	.7	29.2	SHEAR
9	2.3	544.3	27.74	1.73	-68.7	-5.680	-1.307	.9	22.9	SHEAR
10	3.5	543.7	19.91	.87	-66.0	-6.554	-2.589	.9	20.3	SHEAR
11	4.7	543.1	16.61	.42	-65.1	-7.106	-3.589	.9	19.0	SHEAR
12	5.9	542.5	14.77	.18	-64.8	-7.474	-4.395	.8	18.2	SHEAR
13	7.1	541.9	13.56	.07	-64.6	-7.715	-5.051	.8	17.6	SHEAR
14	8.4	541.3	12.67	.01	-64.5	-7.856	-5.580	.8	17.2	SHEAR
15	9.6	540.7	11.97	-.01	-64.3	-7.914	-5.998	.8	16.8	SHEAR
16	10.8	540.1	11.43	-.01	-64.1	-7.898	-6.311	.9	16.4	SHEAR
17	12.0	539.4	11.02	-.01	-63.9	-7.815	-6.523	.9	16.2	SHEAR
18	13.2	538.8	10.74	.01	-63.7	-7.668	-6.632	1.0	16.0	SHEAR
19	14.4	538.2	10.64	.05	-63.3	-7.459	-6.631	1.0	N.A	NO
20	15.7	537.6	10.84	.12	-62.7	-7.190	-6.507	1.1	N.A	NO
21	16.9	537.0	11.58	.25	-61.6	-6.861	-6.235	1.1	N.A	NO
22	18.1	536.4	13.44	.51	-59.4	-6.479	-5.776	1.2	N.A	NO
23	19.3	535.8	17.02	.95	-56.6	-6.059	-5.079	1.1	N.A	NO
24	20.0	535.4	24.15	-2.51	-34.3	-5.794	-4.448	.2	0.0	TENSILE
25	20.3	535.2	24.39	1.09	-28.3	-5.626	-4.190	.8	22.0	SHEAR
26	20.4	534.9	25.80	-.42	-24.1	-5.471	-4.010	.0	23.0	SHEAR
27	20.4	534.2	23.35	2.44	-7.0	-5.069	-3.748	1.2	N.A	NO
28	20.4	533.1	23.14	2.04	-.3	-4.152	-3.405	1.1	N.A	NO
29	20.4	532.0	30.86	2.34	6.7	-2.713	-2.968	.9	23.7	SHEAR
30	20.4	530.8	65.79	5.17	21.7	-.366	-2.129	.6	30.1	SHEAR
31	20.4	530.1	100.67	36.84	56.1	1.906	-1.421	1.3	N.A	NO
32	20.1	529.8	92.72	60.77	56.8	2.651	-1.240	2.0	N.A	NO
33	19.7	529.7	100.31	37.59	59.0	3.093	-.684	1.3	N.A	NO
34	18.9	530.1	60.66	4.98	-83.6	4.763	1.162	.7	29.4	SHEAR
35	17.7	530.7	24.72	1.88	-69.7	6.431	3.090	1.0	N.A	NO
36	16.5	531.3	16.44	.98	-66.4	7.210	4.400	1.2	N.A	NO
37	15.3	531.9	13.16	.49	-65.3	7.644	5.395	1.2	N.A	NO
38	14.1	532.5	11.46	.23	-64.9	7.887	6.177	1.1	N.A	NO
39	12.9	533.1	10.40	.11	-64.7	8.001	6.798	1.1	N.A	NO
40	11.6	533.7	9.66	.05	-64.5	8.017	7.286	1.1	N.A	NO
41	10.4	534.3	9.11	.03	-64.3	7.951	7.658	1.1	N.A	NO
42	9.2	534.9	8.70	.02	-64.1	7.815	7.923	1.2	N.A	NO
43	8.0	535.6	8.42	.03	-63.8	7.613	8.086	1.2	N.A	NO
44	6.8	536.2	8.26	.05	-63.5	7.352	8.146	1.3	N.A	NO
45	5.6	536.8	8.29	.09	-63.0	7.032	8.098	1.4	N.A	NO
46	4.3	537.4	8.61	.16	-62.3	6.656	7.928	1.4	N.A	NO
47	3.1	538.0	9.48	.28	-61.0	6.226	7.616	1.4	N.A	NO
48	1.9	538.6	11.52	.52	-58.4	5.749	7.121	1.4	N.A	NO

49	.7	539.2	15.67	.99	-54.9	5.246	6.392	1.2	N.A	NO
50	-.0	539.6	23.88	-2.34	-31.8	4.936	5.741	.2	0.0	TENSILE
51	-.3	539.8	24.48	1.54	-25.2	4.754	5.479	1.0	21.8	SHEAR
52	-.4	540.1	26.19	.36	-22.1	4.597	5.297	.5	22.8	SHEAR
53	29.6	525.9	22.84	2.39	-7.1	5.043	3.836	1.2	N.A	NO
54	29.6	527.0	22.67	2.02	-.3	4.142	3.503	1.2	N.A	NO
55	29.6	528.1	30.38	2.32	6.7	2.721	3.076	.9	23.5	SHEAR
56	29.6	529.2	65.14	5.11	21.7	.395	2.249	.6	30.0	SHEAR
57	29.6	529.9	99.93	36.48	56.2	-1.862	1.550	1.3	N.A	NO
58	29.9	530.2	92.05	60.26	56.9	-2.604	1.373	2.0	N.A	NO
59	30.3	530.3	99.57	37.32	59.1	-3.042	.825	1.3	N.A	NO
60	31.1	529.9	60.21	4.94	-83.6	-4.695	-1.001	.7	29.3	SHEAR
61	32.3	529.3	24.47	1.86	-69.7	-6.341	-2.905	1.0	N.A	NO
62	33.5	528.7	16.22	.97	-66.4	-7.104	-4.193	1.2	N.A	NO
63	34.7	528.1	12.94	.49	-65.3	-7.524	-5.167	1.2	N.A	NO
64	35.9	527.5	11.22	.23	-64.9	-7.755	-5.930	1.1	N.A	NO
65	37.1	526.9	10.15	.11	-64.7	-7.858	-6.532	1.1	N.A	NO
66	38.4	526.3	9.39	.05	-64.5	-7.863	-7.002	1.1	N.A	NO
67	39.6	525.7	8.83	.03	-64.3	-7.789	-7.357	1.2	N.A	NO
68	40.8	525.1	8.41	.02	-64.0	-7.645	-7.607	1.2	N.A	NO
69	42.0	524.4	8.11	.03	-63.8	-7.438	-7.756	1.3	N.A	NO
70	43.2	523.8	7.94	.05	-63.4	-7.173	-7.804	1.4	N.A	NO
71	44.4	523.2	7.94	.09	-63.0	-6.851	-7.746	1.4	N.A	NO
72	45.7	522.6	8.23	.15	-62.2	-6.475	-7.570	1.5	N.A	NO
73	46.9	522.0	9.05	.27	-60.9	-6.047	-7.256	1.5	N.A	NO
74	48.1	521.4	11.00	.50	-58.3	-5.575	-6.765	1.4	N.A	NO
75	49.3	520.8	14.99	.95	-54.8	-5.077	-6.046	1.3	N.A	NO
76	50.0	520.4	23.00	-2.34	-31.7	-4.770	-5.408	.2	0.0	TENSILE
77	50.3	520.2	23.61	1.44	-25.0	-4.590	-5.152	1.0	21.5	SHEAR
78	50.4	519.9	25.28	.35	-21.9	-4.433	-4.973	.6	22.9	SHEAR
79	50.4	519.2	23.95	2.16	-6.4	-4.087	-4.702	1.1	N.A	NO
80	50.4	518.1	23.24	1.64	-1.1	-3.369	-4.337	1.0	N.A	NO
81	50.4	517.0	28.58	2.01	4.8	-2.273	-3.913	.9	23.1	SHEAR
82	50.4	515.9	56.26	5.09	20.6	-.446	-3.180	.7	28.7	SHEAR
83	50.4	515.1	86.35	32.30	57.3	1.386	-2.587	1.4	N.A	NO
84	50.1	514.8	80.36	53.77	62.7	1.994	-2.437	2.1	N.A	NO
85	49.7	514.7	87.25	35.43	62.0	2.378	-1.961	1.5	N.A	NO
86	48.9	515.1	57.34	4.87	-81.8	3.915	-.338	.7	28.9	SHEAR
87	47.7	515.7	26.62	1.66	-68.7	5.535	1.418	.9	22.5	SHEAR
88	46.5	516.3	19.14	.84	-66.0	6.384	2.656	1.0	20.0	SHEAR
89	45.3	516.9	16.00	.40	-65.2	6.925	3.625	.9	18.7	SHEAR
90	44.1	517.5	14.27	.17	-64.8	7.289	4.409	.9	18.0	SHEAR
91	42.9	518.1	13.12	.06	-64.7	7.531	5.048	.8	17.4	SHEAR
92	41.6	518.7	12.29	.01	-64.5	7.677	5.567	.8	16.9	SHEAR
93	40.4	519.3	11.65	-.01	-64.3	7.743	5.978	.9	16.6	SHEAR
94	39.2	519.9	11.15	-.02	-64.2	7.737	6.290	.9	16.3	SHEAR
95	38.0	520.6	10.77	-.01	-64.0	7.666	6.503	.9	16.0	SHEAR
96	36.8	521.2	10.52	.01	-63.7	7.532	6.617	1.0	15.8	SHEAR
97	35.6	521.8	10.45	.05	-63.4	7.337	6.623	1.0	N.A	NO
98	34.3	522.4	10.67	.12	-62.8	7.083	6.509	1.1	N.A	NO
99	33.1	523.0	11.41	.24	-61.7	6.771	6.251	1.1	N.A	NO
100	31.9	523.6	13.26	.50	-59.4	6.406	5.808	1.2	N.A	NO
101	30.7	524.2	16.78	.93	-56.7	6.003	5.133	1.1	N.A	NO
102	30.0	524.6	23.72	-2.47	-34.5	5.748	4.519	.2	0.0	TENSILE
103	29.7	524.8	23.93	1.04	-28.5	5.585	4.267	.8	21.8	SHEAR
104	29.6	525.1	25.30	-.48	-24.2	5.435	4.091	.9	0.0	TENSILE

NN = 2

STRESSES AND DISPLACEMENTS AND FAILURE CRITERION AT INTERIOR POINTS

I	CX (M)	CZ (M)	SIG1 (MAPA)	SIG3 (MAPA)	ALPHA (DEG)	UX (MM)	UZ (MM)	F.O.S	BETA (DEG)	FAILURE
1	-.8	540.9	24.87	3.07	-8.6	4.047	4.716	1.3	N.A	NO
2	-.8	542.0	26.81	2.48	-.2	3.308	4.304	1.1	N.A	NO
3	-.8	543.1	34.17	3.07	10.0	2.180	3.783	.9	24.4	SHEAR
4	-.8	544.2	56.11	8.60	31.2	.391	3.057	1.0	28.0	SHEAR
5	-.7	545.0	67.72	28.73	63.8	-1.248	2.658	1.7	N.A	NO
6	-.3	545.5	60.12	45.75	86.7	-1.916	2.510	2.5	N.A	NO
7	.3	545.7	67.84	31.47	56.4	-2.297	1.932	1.8	N.A	NO
8	1.2	545.3	57.87	6.98	87.9	-3.507	.359	.8	28.6	SHEAR
9	2.5	544.7	31.91	1.44	-74.1	-5.147	-1.400	.7	24.2	SHEAR
10	3.7	544.1	22.75	.61	-69.5	-6.116	-2.642	.7	21.5	SHEAR
11	4.9	543.5	18.65	.31	-67.8	-6.739	-3.612	.7	20.0	SHEAR
12	6.1	542.8	16.37	.16	-67.0	-7.161	-4.395	.7	19.0	SHEAR
13	7.3	542.2	14.88	.09	-66.5	-7.444	-5.032	.8	18.3	SHEAR
14	8.5	541.6	13.83	.06	-66.0	-7.621	-5.545	.8	17.8	SHEAR
15	9.7	541.0	13.04	.04	-65.4	-7.711	-5.948	.8	17.4	SHEAR
16	11.0	540.4	12.45	.05	-64.9	-7.726	-6.248	.9	17.0	SHEAR
17	12.2	539.8	12.03	.06	-64.3	-7.672	-6.447	.9	16.8	SHEAR
18	13.4	539.2	11.78	.08	-63.6	-7.555	-6.542	.9	16.6	SHEAR
19	14.6	538.6	11.75	.13	-62.7	-7.377	-6.526	1.0	16.5	SHEAR
20	15.8	538.0	12.05	.22	-61.3	-7.141	-6.386	1.0	N.A	NO
21	17.0	537.4	12.91	.39	-59.1	-6.849	-6.096	1.1	N.A	NO
22	18.3	536.8	14.79	.79	-55.1	-6.503	-5.619	1.2	N.A	NO
23	19.5	536.1	18.04	1.33	-49.3	-6.107	-4.912	1.2	N.A	NO
24	20.3	535.7	22.53	.82	-34.0	-5.795	-4.258	.8	21.4	SHEAR
25	20.6	535.4	23.41	2.56	-26.5	-5.568	-3.926	1.3	N.A	NO
26	20.8	534.9	24.47	2.81	-20.3	-5.324	-3.674	1.3	N.A	NO
27	20.8	534.2	24.70	3.44	-8.9	-4.872	-3.397	1.4	N.A	NO
28	20.8	533.1	27.13	3.02	1.6	-3.942	-2.986	1.2	N.A	NO
29	20.8	532.0	36.86	3.18	12.9	-2.495	-2.437	.9	25.1	SHEAR
30	20.8	530.8	63.90	7.92	32.6	-.254	-1.630	.8	29.4	SHEAR
31	20.7	530.0	75.96	30.64	61.5	1.770	-1.167	1.5	N.A	NO
32	20.3	529.5	66.68	48.74	55.0	2.614	-.998	2.4	N.A	NO
33	19.7	529.3	75.67	30.61	52.3	3.041	-.364	1.5	N.A	NO
34	18.8	529.7	59.78	6.40	85.2	4.268	1.339	.8	29.0	SHEAR
35	17.5	530.3	29.54	1.46	-75.7	5.903	3.194	.8	23.5	SHEAR
36	16.3	530.9	19.57	.67	-70.3	6.788	4.458	.9	20.2	SHEAR
37	15.1	531.5	15.35	.36	-68.4	7.300	5.419	.9	18.4	SHEAR
38	13.9	532.2	13.13	.20	-67.4	7.599	6.176	1.0	17.3	SHEAR
39	12.7	532.8	11.76	.13	-66.7	7.757	6.776	1.0	16.6	SHEAR
40	11.5	533.4	10.83	.09	-66.0	7.809	7.247	1.0	N.A	NO
41	10.3	534.0	10.17	.08	-65.4	7.776	7.604	1.1	N.A	NO
42	9.0	534.6	9.70	.08	-64.7	7.669	7.855	1.1	N.A	NO
43	7.8	535.2	9.40	.09	-63.9	7.497	8.005	1.2	N.A	NO
44	6.6	535.8	9.26	.11	-63.0	7.264	8.051	1.2	N.A	NO
45	5.4	536.4	9.35	.15	-61.9	6.974	7.987	1.3	N.A	NO
46	4.2	537.0	9.76	.23	-60.2	6.630	7.802	1.3	N.A	NO
47	3.0	537.6	10.75	.38	-57.7	6.234	7.472	1.3	N.A	NO
48	1.7	538.2	12.83	.72	-53.2	5.792	6.960	1.4	N.A	NO

49	.5	538.9	16.57	1.25	-46.4	5.306	6.224	1.3	N.A	NO
50	-.3	539.3	21.55	.79	-30.4	4.938	5.562	.8	21.0	SHEAR
51	-.6	539.6	22.61	2.61	-22.8	4.690	5.236	1.3	N.A	NO
52	-.8	540.1	23.91	2.91	-17.8	4.443	4.993	1.3	N.A	NO
53	29.2	525.9	24.24	3.37	-9.1	4.849	3.492	1.4	N.A	NO
54	29.2	527.0	26.67	2.99	1.6	3.935	3.089	1.2	N.A	NO
55	29.2	528.1	36.38	3.16	12.9	2.506	2.550	.9	25.0	SHEAR
56	29.2	529.2	63.35	7.83	32.7	.283	1.754	.8	29.4	SHEAR
57	29.3	530.0	75.47	30.33	61.5	-1.729	1.297	1.5	N.A	NO
58	29.7	530.5	66.26	48.33	55.1	-2.570	1.133	2.4	N.A	NO
59	30.3	530.7	73.15	30.39	52.3	-2.994	.508	1.5	N.A	NO
60	31.2	530.3	59.35	6.35	85.1	-4.207	-1.175	.8	29.0	SHEAR
61	32.5	529.7	29.27	1.45	-75.7	-5.820	-3.005	.8	23.5	SHEAR
62	33.7	529.1	19.33	.67	-70.3	-6.689	-4.248	.9	20.1	SHEAR
63	34.9	528.5	15.12	.36	-68.3	-7.186	-5.189	.9	18.3	SHEAR
64	36.1	527.8	12.88	.20	-67.3	-7.473	-5.927	1.0	17.2	SHEAR
65	37.3	527.2	11.50	.13	-66.6	-7.620	-6.509	1.0	N.A	NO
66	38.5	526.6	10.56	.09	-65.9	-7.662	-6.962	1.1	N.A	NO
67	39.7	526.0	9.88	.08	-65.3	-7.620	-7.303	1.1	N.A	NO
68	41.0	525.4	9.40	.08	-64.6	-7.506	-7.539	1.2	N.A	NO
69	42.2	524.8	9.07	.09	-63.8	-7.328	-7.675	1.2	N.A	NO
70	43.4	524.2	8.92	.11	-62.9	-7.091	-7.710	1.3	N.A	NO
71	44.6	523.6	8.97	.15	-61.7	-6.798	-7.638	1.3	N.A	NO
72	45.8	523.0	9.35	.22	-60.0	-6.454	-7.446	1.4	N.A	NO
73	47.0	522.4	10.29	.36	-57.5	-6.059	-7.115	1.4	N.A	NO
74	48.3	521.8	12.29	.68	-52.9	-5.620	-6.607	1.4	N.A	NO
75	49.5	521.1	15.89	1.17	-46.2	-5.140	-5.882	1.3	N.A	NO
76	50.3	520.7	20.78	.69	-30.1	-4.777	-5.233	.8	20.7	SHEAR
77	50.6	520.4	21.83	2.46	-22.5	-4.531	-4.913	1.3	N.A	NO
78	50.8	519.9	23.11	2.80	-17.5	-4.285	-4.675	1.3	N.A	NO
79	50.8	519.2	24.06	2.98	-8.3	-3.890	-4.405	1.3	N.A	NO
80	50.8	518.1	25.95	2.39	-.1	-3.161	-4.004	1.1	N.A	NO
81	50.8	517.0	33.02	2.94	10.0	-2.060	-3.497	.9	24.1	SHEAR
82	50.8	515.9	54.07	8.27	31.2	-.326	-2.792	1.0	27.7	SHEAR
83	50.7	515.0	65.07	27.68	63.7	1.256	-2.404	1.7	N.A	NO
84	50.3	514.5	57.72	44.03	66.3	1.900	-2.260	2.6	N.A	NO
85	49.7	514.3	65.20	30.19	56.3	2.269	-1.701	1.8	N.A	NO
86	48.8	514.7	55.56	6.68	87.8	3.437	-.187	.9	28.3	SHEAR
87	47.5	515.3	30.62	1.37	-74.1	5.022	1.509	.7	23.9	SHEAR
88	46.3	515.9	21.86	.58	-69.5	5.962	2.709	.7	21.2	SHEAR
89	45.1	516.5	17.95	.29	-67.9	6.571	3.649	.7	19.7	SHEAR
90	43.9	517.2	15.79	.15	-67.1	6.986	4.410	.8	18.8	SHEAR
91	42.7	517.8	14.39	.08	-66.5	7.268	5.032	.8	18.1	SHEAR
92	41.5	518.4	13.41	.05	-66.0	7.448	5.535	.8	17.6	SHEAR
93	40.3	519.0	12.67	.04	-65.5	7.545	5.932	.8	17.2	SHEAR
94	39.0	519.6	12.13	.04	-65.0	7.568	6.230	.9	16.8	SHEAR
95	37.8	520.2	11.75	.06	-64.4	7.525	6.430	.9	16.6	SHEAR
96	36.6	520.8	11.53	.08	-63.7	7.419	6.530	1.0	16.4	SHEAR
97	35.4	521.4	11.53	.13	-62.8	7.255	6.522	1.0	N.A	NO
98	34.2	522.0	11.85	.21	-61.4	7.034	6.392	1.1	N.A	NO
99	33.0	522.6	12.72	.39	-59.2	6.757	6.115	1.1	N.A	NO
100	31.7	523.2	14.58	.79	-55.2	6.427	5.655	1.2	N.A	NO
101	30.5	523.9	17.78	1.32	-49.5	6.047	4.970	1.2	N.A	NO
102	29.7	524.3	22.16	.81	-34.2	5.746	4.333	.8	21.2	SHEAR
103	29.4	524.6	23.01	2.49	-26.7	5.527	4.009	1.3	N.A	NO
104	29.2	525.1	24.04	2.72	-20.5	5.290	3.763	1.3	N.A	NO